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An inventory model for deteriorating items of two parameter weibull distribution with stock dependent demand and money inflation

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Abstract
This paper deals with an inventory model in which shortages are allowed and partially backlogged. A two-parameter Weibull distribution is taken to represent the time to deterioration. It is also assumed that the demand is a function of stock and the money value is subject to inflation.

Keywords: Inflation, weibull distribution, stock-dependent demand, deterioration, shortages, partial backlogging

Introduction

In most of the inventory models, the demand rate is considered to be either constant or time dependent but independent of stock status. However, in the competitive market situation customers are influenced by the marketing policies such as the attractive display of units in the market or the business place. As pointed out by Levin *et al*. [2], at times, the presence of inventory has a motivational effect on the people around it and attracts the people to buy more.

A good number of authors have studied problems connected with inventories of stock-level dependent demand rate. Some of the recent studies of such models have been carried out in the research articles published by Gupta and Vrat [7], Mandal and Phaujdar [9], Datta and Pal [12] etc.

Gupta and Vrat [7] have discussed the models in which demand rate has been assumed to depend upon the order quantity. Datta and Pal [12] have extended the model to one in which the demand rate depends upon inventory level down to a certain stock level and then becomes constant.

Many mathematical models have been developed for controlling the inventory. In several exciting models, it is assumed that the products have infinite shelf time. But actually deterioration plays a vital role in inventory. Deterioration is defined as decay, spoilage, loss of utility of products etc. The process of deterioration is observed in volatile liquids, beverages, medicines, blood components, sweets, fruits and vegetables etc., that result in decrease of usefulness of the original one. In recent years, inventory problems for deteriorating items have been widely studied after Ghare and Schrader [1]. Philip [3] extended Ghare and Schrader [1] model with a three parameter Weibull distribution rate and no shortages. Deb and Chaudhari [8] derived inventory models with time dependent deterioration rate.

In market, it is observed that because of good reputation of the retailer, some customers are willing to wait for new stocks arrival, or if the wait will be short, while other may go elsewhere. Researchers, such as Park [6], Wee [10] developed inventory models with partial backorders.

Inflation is a concept closely related to time. Inflation is generally associated with rapidly rising prices which causes or are caused by a decline in the purchasing power of money which varies or rather depends upon time. In recent years, while determining the optimal inventory policy, the effects of inflation and time value of money cannot be ignored. The investigation of inflation that took off with Buzacott [4], saw of the likes of Misra [5], Dutta and Pal [12], and Bose *et al.* [11] further extended the concept of inflation. Recently, Wu *et al.* [14] developed a replenishment policy for non-instantaneous deteriorating items with stock-dependent demand and partial backlogging. In this article we have developed an inventory model with stock-dependent demand rate assuming the life-time of the perishable items to be random and follow two parameter Weibull distributions. Numerical examples are presented to demonstrate the developed model and to illustrate the procedure.

Assumption and Notations

1. The replenishment occurs instantaneously at an infinite rate.
2. Lead time is zero.
3. The life time of the commodity is random and follows a two parameter Weibull distribution. Hence the instantaneous rate of deterioration of on hand inventory is $h(t) = \alpha\beta\tau^{\beta-1}$
4. The demand rate D(t) at time 't' is

$$D(t)\begin{cases}\tau+\phi q(t) \quad ; \quad q(t)>0\\ \tau \quad ; \quad q(t)\le 0\end{cases}$$

where τ and ϕ are positive constants. If ϕ = 0, then the demand rate is constant.

5. Shortages are allowed and partially backlogged.
6. During time t_1, the inventory has no shortage, t_1' is the length of time in which product has no deterioration.
7. Q is the order quantity per cycle and T is the length of order cycle.
8. There is no repair or replenishment of deteriorated units.
9. C_0 denotes the ordering cost per order, C_h is the inventory holding cost coefficient per unit time, C_d deteriorating cost per unit, C_s is the shortage cost for backlogged items and □ is the unit cost of lost sales. All the cost parameters are positive constants.
10. $q_1(t)$ denotes the inventory level at time t $(0\le t\le t_1')$ in

    which the product has no deterioration, $q_2(t)$ is the inventory level at time t $(t_1'\le t\le t_1)$ in which the product has deterioration. $q_3(t)$ denotes the inventory level at time t $(t_1\le t\le T)$ in which the product has shortage.
11. 'r' is a constant representing the difference between the discount rate and inflation rate.
12. The model has been developed for a finite planning horizon.

Mathematical model

The differential equations describing the instantaneous states of q (t) in the interval (0, T) are given by

$$\frac{dq_1(t)}{dt}=-(\tau+\phi q_1(t)); \qquad 0\le t\le t_1' \qquad \text{... (1)}$$

$$\frac{dq_2(t)}{dt}+\alpha\beta t^{\beta-1}q_2(t)=-[\tau+\phi q_2(t)]; \quad t_1'\le t\le t_1 \qquad \text{... (2)}$$

$$\frac{dq_3(t)}{dt}=-\tau\delta; \qquad t_1\le t\le T \qquad \text{... (3)}$$

The solutions of the above differential equations after applying the boundary conditions $q_1(0) = q_m$; $q_2(t_1) = 0$; $q_3(t_1) = 0$ are

$$q_1(t)=e^{-\phi t}q_m-\frac{\tau}{\phi}[1-e^{-\phi t}]; \quad 0\le t\le t_1' \qquad \text{... (4)}$$

$$q_2(t)=\tau\left[(t_1-t)+\frac{\alpha}{\beta+1}(t_1^{\beta+1}-t^{\beta+1})+\frac{\phi}{2}(t_1^2-t^2)\right]e^{-(\alpha t^\beta+\phi t)}; \quad t_1'\le t\le t_1 \qquad \text{... (5)}$$

$$q_3(t)=-\delta\tau(t-t_1) \qquad ; \qquad t_1\le t\le T \qquad \text{... (6)}$$

Taking into consideration the continuity of q(t) at t = t_1', it follows that $q_1(t_1')=q_2(t_1')$ which implies that the maximum inventory level for each cycle is

$$q_m=\tau\left[(t_1-t_1')+\frac{\alpha}{\beta+1}(t_1^{\beta+1}-t_1'^{\beta+1})+\frac{\phi}{2}(t_1^2-t_1'^2)\right]e^{-\alpha t_1'^\beta}+\frac{\tau}{\phi}\left[e^{\phi t_1'}-1\right] \qquad \text{...(7)}$$

Substituting equation (7) in (4), we get

$$q_1(t)=\tau\left[\{(t_1-t_1')+\frac{\alpha}{\beta+1}(t_1^{\beta+1}-t_1'^{\beta+1})+\frac{\phi}{2}(t_1^2-t_1'^2)\}e^{-(\alpha t_1'^\beta+\phi t)}\right]+\frac{\tau}{\phi}\left[e^{-\phi(t-t_1')}-1\right] \qquad \text{...(8)}$$

The maximum amount of demand backlogged per cycle is given by

$$q_b = -q_3(T) = \delta\tau(T - t_1) \qquad \text{... (9)}$$

Using equation (7) and (9), we have order quantity Q, as

$$Q = q_m + q_b$$

$$= \tau\left[(t_1 - t_1') + \frac{\alpha}{\beta+1}\left(t_1^{\beta+1} - t'^{\beta+1}\right) + \frac{\phi}{2}\left(t_1^2 - t_1'^2\right)\right] e^{-\alpha t_1'^{\beta}} + \frac{\tau}{\phi}\left[e^{\phi t_1'} - 1\right] + \delta\tau(T - t_1) \qquad \text{…(10)}$$

The total cost per cycle consists of the following cost components:

1. Since the order for replenishing the stock is placed at the beginning of each cycle, the value of ordering cost per cycle is

$$OC = C_o \qquad \text{... (11)}$$

2. Inventory holding cost per cycle is given by

$$IHC = C_h\int_0^{t_1'} q_1(t)e^{-rt}dt + C_h\int_{t_1'}^{t_1} q_2(t)e^{-rt}dt$$

$$IHC = \tau C_h\Bigg[\left\{\left(t_1 - t_1'\right) + \frac{\alpha}{\beta+1}\left(t_1^{\beta+1} - t_1'^{\beta+1}\right) + \frac{\phi}{2}\left(t_1^2 - t_1'^2\right)\right\}\frac{e^{-\alpha t_1'^{\beta}}}{(\phi+r)}\left\{1 - e^{-(\phi+r)t_1'}\right\}$$

$$+\frac{1}{\phi}\left\{\frac{(e^{\phi t_1'} - e^{-rt_1'})}{(\phi+r)} + \frac{1}{r}(e^{-rt_1'} - 1)\right\} + \frac{t_1^2}{2} + \frac{\alpha\beta}{(\beta+1)(\beta+2)}t_1^{\beta+2} + \frac{\phi}{6}t_1^3 - \frac{r}{6}t_1^3 - t_1t_1' + \frac{t_1'^2}{2}$$

$$+\frac{\alpha t_1 t_1'}{\beta+1}\left(t_1'^{\beta} - t_1^{\beta}\right) - \frac{\alpha\beta}{(\beta+1)(\beta+2)}t_1'^{\beta+2} + \frac{\phi}{2}t_1t_1'(t_1' - t_1) - \frac{\phi}{6}t_1'^3 + \frac{\alpha\phi}{2(\beta+1)}t_1t_1'\,(t_1t_1'^{\beta} + t_1't_1^{\beta})$$

$$+\frac{rt_1t_1'^2}{2} - \frac{rt_1'^3}{3} + \frac{\phi^2}{4}t_1^2t_1'^2 + \frac{r\phi}{4}t_1^2t_1'^2\Bigg] \qquad \text{... (12)}$$

3. Deterioration cost per cycle is given by

$$DC = C_d\int_{t_1'}^{t_1} q_2(t)e^{-rt}dt$$

$$= C_d\tau\Bigg[\frac{t_1^2}{2} + \frac{\alpha\beta}{(\beta+1)(\beta+2)}t_1^{\beta+2} + \frac{\phi}{6}t_1^3 - \frac{r}{6}t_1^3 - t_1t_1' + \frac{t_1'^2}{2} - \frac{\alpha}{(\beta+1)}t_1^{\beta+1}t_1' - \frac{\alpha\beta}{(\beta+1)(\beta+2)}t_1'^{\beta+2} - \frac{\phi}{2}t_1^2t_1'$$

$$-\frac{\phi}{6}t_1'^3 + \frac{\alpha}{\beta+1}t_1t_1'^{\beta+1} + \frac{\alpha\phi}{2}\frac{t_1^2}{(\beta+1)}t_1'^{\beta+1} + \frac{(\phi+r)}{2}t_1t_1'^2 - \frac{r}{3}t_1'^3 + \frac{\alpha}{2}\frac{(\phi+r)}{(\beta+1)}t_1^{\beta+1}t_1'^2 + \frac{\phi}{4}t_1^2t_1'^2(\phi+r)\Bigg] \text{...(13)}$$

4. Shortage cost per cycle due to backlog is given by

$$SC = C_s\int_{t_1}^{T}\left[-q_3(t)\right]e^{-rt}dt = \tau\delta\frac{C_s}{r^2}\left[e^{-rt_1} - e^{-rT}\left\{r(T - t_1) + 1\right\}\right] \qquad \text{…(14)}$$

5. Opportunity cost per cycle due to lost sales is given by

$$LS = \pi\int_{t_1}^{T}\tau(1-\delta)e^{-rt}dt = \frac{\pi\tau(1-\delta)}{r}\left[e^{-rt_1} - e^{-rT}\right] \qquad \text{…(15)}$$

6. Purchase cost per cycle is given by

$$PC = pq_m + pe^{-rT}q_b$$

$$PC = p\tau\left[\left\{(t_1 - t_1') + \frac{\alpha}{\beta+1}\left(t_1^{\beta+1} - t_1'^{\beta+1}\right) + \frac{\phi}{2}\left(t_1^2 - t_1'^2\right)\right\} e^{-\alpha t_1'^{\beta}} + \frac{1}{\phi}\left(e^{\phi t_1'} - 1\right)\right] + p\delta\tau(T - t_1)e^{-rT} \quad ...(16)$$

Therefore, total inventory cost per unit time is given by

$$TC(t_1, T) = \frac{1}{T}[OC + IHC + DC + SC + LS + PC]$$

Solution procedure
Now the necessary condition for the total cost per unit time to be minimum is

$$\frac{\partial TC}{\partial t_1}(t_1, T) = 0 \quad \text{and} \quad \frac{\partial TC}{\partial T}(t_1, T) = 0$$

provided they satisfy the sufficient conditions

$$\left.\frac{\partial^2 TC(t_1,T)}{\partial t_1^2}\right|_{(t_1^*,T^*)} > 0, \quad \left.\frac{\partial^2 TC(t_1,T)}{\partial T^2}\right|_{(t_1^*,T^*)} > 0 \quad \left\{\left(\frac{\partial^2 TC}{\partial t_1^2}(t_1,T)\right)\left(\frac{\partial^2 TC}{\partial T^2}(t_1,T)\right) - \left(\frac{\partial^2 TC}{\partial t_1 \partial T}(t_1,T)\right)^2\right\}_{(t_1^*,T^*)} > 0$$

Numerical Example

Table 1: Relation between 'α' and 'TC'

t'_1=0.0833, C_s=2.5	β=2,	r=0.52	C_0 =250,	C_h =0.5,	C_d =1.5,		δ=0.56,	p=2

τ=600, φ=0.7 π=2

Table 1.1

α	t_1	T	TC	Q
0.02	0.1382	0.73138883	1849.566	286.2743
0.04	0.1381	0.73099404	1848.389	286.1128
0.06	0.1381	0.73123347	1848.398	286.1965
0.08	0.138	0.73083418	1848.428	286.0334
0.1	0.138	0.7310719	1848.438	286.1166
0.12	0.138	0.73130945	1848.447	286.1997
0.14	0.1379	0.73090412	1848.477	286.0345
0.16	0.1379	0.73113996	1848.486	286.117
0.18	0.1379	0.73137564	1848.496	286.1994

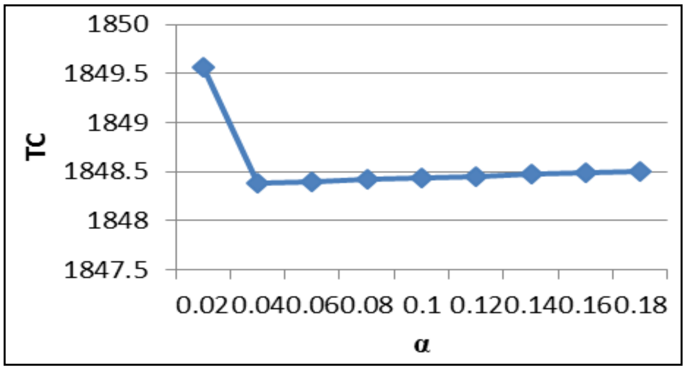

Fig 1

When ‘α’ increases, the inventory period has fluctuations and total inventory cost decreases.

Table 2: Relation between ‘r’ and ‘TC’

t'_1=0.0833 C_s=2.5	β=2,	α=0.1	C_0 =250,	C_h =0.5,	C_d =1.5,		δ=0.56,	p=2

τ=600, φ=0.7 π=2

Table 2.1

r	t_1	T	TC	Q
0.44	0.1462	0.69638293	1878.613	277.1215
0.46	0.1442	0.70476593	1870.393	279.2866
0.48	0.1421	0.71293026	1863.233	281.3475
0.5	0.1401	0.722097	1855.903	283.7795
0.52	0.138	0.7310719	1848.438	286.1166
0.54	0.1359	0.74049315	1840.811	288.6055
0.56	0.1339	0.75107874	1832.999	291.5197
0.58	0.1318	0.76153305	1825.033	294.3595
0.6	0.1297	0.77256336	1816.887	297.3948

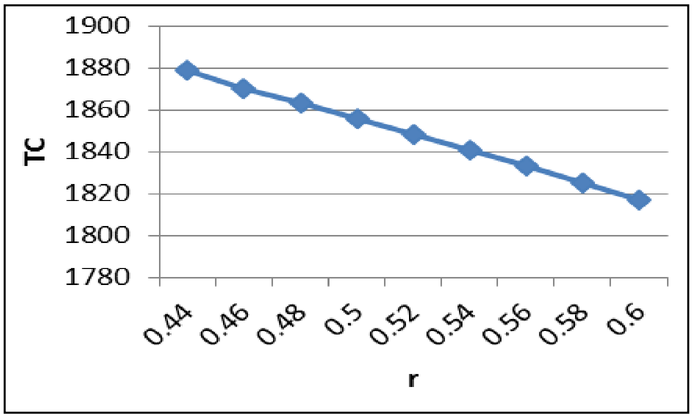

Fig 2

When ‘r’ increases, the inventory period increases and total inventory cost decreases.

Table 3: Relation between ‘ϕ’ and ‘TC’

t'_1=0.0833, C_s=2.5	β=2,	α=0.1	C_0 =250,	C_h =0.5,	C_d =1.5,		δ=0.56,	p=2

τ=600, r=0.52 π=2

Table 3.1

φ	t_1	T	TC	Q
0.62	0.1424	0.73101034	1847.39	287.0271
0.64	0.1413	0.73119433	1846.915	286.8609
0.66	0.1402	0.73126577	1847.431	286.6536
0.68	0.1391	0.73122489	1847.938	286.4055
0.7	0.138	0.7310719	1848.438	286.1166
0.72	0.137	0.73145195	1848.909	286.0361
0.74	0.1359	0.73108035	1849.393	285.6678
0.76	0.1349	0.73125217	1849.85	285.5117
0.78	0.1339	0.73132263	1850.299	285.3188

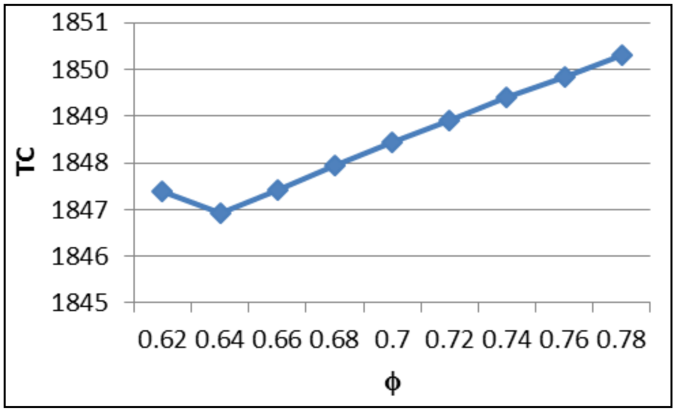

Fig 3

When ‘φ’ increases, the inventory period increases and total inventory cost also increases.

Table 4: Relation between ‘τ’ and ‘TC’

t'_1=0.0833, C_s=2.5	β=2,	α =0.1,	C_0 =250,	C_h =0.5,	C_d =1.5,		δ=0.56,	p=2

φ=0.7, π=2, r=0.52

Table 4.1

τ	t_1	T	TC	Q
500	0.1489	0.80083324	1593.880849	260.916
525	0.1459	0.78163185	1657.329893	267.4588
550	0.1431	0.76371117	1721.349075	273.84
575	0.1405	0.74707119	1785.039422	280.1211
600	0.138	0.7310719	1848.437597	286.1166
625	0.1357	0.71635322	1911.540113	292.1148
650	0.1335	0.70227517	1974.3812	297.9095
675	0.1315	0.68947757	2036.951787	303.8095

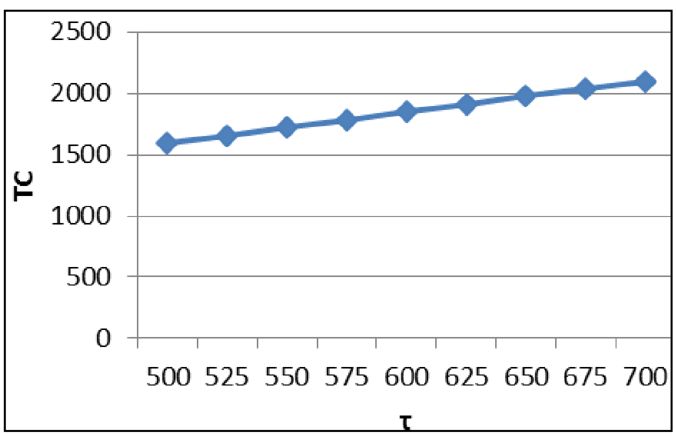

Fig 4

When ‘τ’ increases, the inventory period decreases and total inventory cost increases.

Conclusion

In this paper, we have proposed inventory models for perishable items having stock dependent demand and Weibull decay. The two parameter Weibull distribution considered in this study can be applied for the items with any initial value of the rate of deterioration and for items which start deteriorating only after a certain period of time. The extent to which inflation has affected the business world is clearly elucidated. Our research implies that the effect of inflation and time value of money on the present value of total cost is more significant and highlights that total cost decreases as the inflation rate increases.

Thus, this model incorporates some realistic features that are likely being associated with some kinds of inventory.

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